Self-consistent mode-coupling approach to the nonlocal Kardar-Parisi-Zhang equation

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The dynamic scaling of the nonlocal Kardar-Parisi-Zhang equation in the strong-coupling regime is investigated by a self-consistent mode-coupling approximation. The values of the dynamic exponent depending on nonlocal parameter ρ are calculated numerically for the substrate dimension $d=1$, $d=2$, and $d=3$, respectively.

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The Kardar-Parisi-Zhang (KPZ) equation [1] is one of the most prominent models describing nontrivial nonequilibrium dynamics and has attracted much attention in recent years [2]. In addition to the coarse-grained description of a wide variety of growth processes, such as the Eden model, growth by ballistic deposition, and the growth of an interface in random medium [2], it is also related to many other important physical problems such as randomly stirred fluids [3] (Burgers equation), dissipative transport in the driven-diffusion equation [4,5], the directed polymer problem in a random potential [6], and the behavior of flux lines in superconductors [7]. So any advance in understanding the KPZ equation will possibly have wide significance both in the fields of nonequilibrium dynamics and in disordered systems.

The KPZ equation for a growing interface is

$$\frac{\partial h(\mathbf{x},t)}{\partial t}=\nu\nabla^2 h+\frac{\lambda}{2}(\nabla h)^2+\eta(\mathbf{x},t). \tag{1}$$

It describes the height fluctuations $h(\mathbf{x},t)$ of a stochastically grown d-dimensional interface with a growth rate $v(\nabla h)=\lambda(\nabla h)^2/2$ depending nonlinearly on the local orientation of the interface. The $(\nu\nabla^2 h)$ term mimics a surface tension, and acts to smooth the interface, while the uncorrelated Langevin noise $\eta(\mathbf{x},t)$ tends to roughen the interface and entails the stochastic nature of a growth process. The noise has zero mean and is Gaussian, such that

$$\langle \eta(\mathbf{x},t)\,\eta(\mathbf{x}',t')\rangle=2D\,\delta^d(\mathbf{x}-\mathbf{x}')\,\delta(t-t'), \tag{2}$$

where d is the substrate dimension and D specifies the noise amplitude [1].

The steady state interface profile is usually described in terms of the roughness, $w(L,t)=\sqrt{\langle h^2(\mathbf{x},t)\rangle-\langle h(\mathbf{x},t)\rangle^2}$, which for a system of size L behaves like $L^{\chi}f(t/L^z)$ [8], where the scaling function $f(u)\to\text{const}$ as $u\to\infty$ and $f(u)\to u^{\chi/z}$ as $u\to 0$, so that $w\sim t^{\chi/z}$ for $t\ll L^z$ and $w\sim L^{\chi}$ for $t\gg L^z$. The scaling exponents χ and z are the roughness and the dynamic exponent, respectively. The phenomenology of the KPZ equation is now well known [9]: Below the lower critical dimension $d_c=2$, there appear two renormalization-group (RG) fixed points, namely, an infrared-unstable Gaussian fixed point and an infrared-stable strong-coupling fixed point describing a smooth and a rough interface, respectively. For substrate dimensions $d>2$ there exists a nonequilibrium phase transition from a weak-coupling phase for small effective coupling constants $g=\lambda^2 D/\nu^3$ to a strong-coupling phase. In the weak-coupling phase, the nonlinear term is irrelevant and the behavior of the KPZ equation is governed by the Gaussian $(\lambda=0)$ fixed point, the KPZ in this phase is equivalent to the linear Edwards-Wilkinson equation, for which the scaling exponents are known exactly to be $\chi=(2-d)/2$ and $z=2$. While in the strong-coupling phase, the nonlinear term is relevant and the scaling relation $\chi+z=2$ (for all d) follows from the the invariance of Eq. (1) to an infinitesimal titling of the surface $h\to h+\mathbf{v}\cdot\mathbf{x}$, $\mathbf{x}\to\mathbf{x}-\lambda\mathbf{v}t$ [10]. It should also be mentioned that the scaling relation $\chi+z=2$ holds for finite renormalization-group fixed points and it is not clear at all that this scaling relation should also apply to the strong-coupling regime beyond the roughening transition for $d\geq 2$. Accordingly, there is only one independent exponent to be determined in the strong-coupling regime. For the special case $d=1$, the existence of a fluctuation-dissipation theorem leads to the exact result $\chi=1/2$ and $z=3/2$. However, the scaling exponents in general dimension d are not known exactly up to now and the behavior of the KPZ is controversial. In addition, it has been shown that if the noise in Eq. (1) is Gaussian spatially long-range correlated noise and characterized by its second moment $R(\mathbf{x}-\mathbf{x}')\propto|\mathbf{x}-\mathbf{x}'|^{2\sigma-d}$, the lower critical dimension for the roughening transition is shifted upwards to $d_c=2+2\sigma$ [10,11]. In order to gain a better understanding of the KPZ equation and possibly reveal some of its hidden secrets, Janssen, Täuber, and Frey [11] recently investigated the KPZ equation in d spatial dimensions with Gaussian spatially long-range correlated noise by means of dynamic field theory and the renormalization group. They fully discussed the scaling regimes and critical dimensions in the KPZ problem and argued that there is an intriguing possibility that the rough phases above and below the lower critical dimension $d_c=2+2\sigma$ are genuinely different.

The KPZ equation and most of its modifications are generally related only to short-range (or local) nature of interaction in the nonlinear term that describes the lateral growth [1]. In many growth problems, however, the long-range interactions, e.g., the long-ranged hydrodynamic interactions, are necessary [12,13]. In order to incorporate these long-range interactions into the kinetic roughening of surface, Mukherji and Bhattacharjee [14] proposed the nonlocal KPZ equation that is a phenomenological equation with a nonlinear term of long-range nature capable of correlating each site of the growing surface with all other sites. By dynamic renormalization-group (DRG) analysis, they show that the nonlocal nonlinearity introduced is sufficient to yield new fixed points with continuously varying exponents depending on the long-range feature, and several distinct phase transitions that were not found in the local KPZ theory. After that, the effects of long-range interaction on the conserved KPZ equation and the noisy Kuramoto-Sivashinsky equation were studied by DRG technique, respectively [15,16].

In the analysis of the dynamic scaling behavior of nonlinear Langevin-type equations, the DRG theory is the most widely used analytical method [1,3,10,14–16]. The DRG theory, however, has had only limited success. This is because in the strong-coupling regime, the exponents are controlled by some nontrivial strong-coupling fixed points that are inaccessible through a perturbative DRG analysis [17]. In the analytical theory, a major theoretical difficulty that in the strong-coupling regime the perturbative series in λ about $\lambda=0$ cannot be summed self-consistently in terms of just response and correlation functions because of vertex correction graphs that renormalize the nonlinearity. The nonperturbative mode-coupling approximation essentially consists in a resummation of the perturbative theory in which all propagator renormalizations are properly taken into account, while the vertex corrections are neglected completely [3,18]. This seems to be a very strange and uncontrolled procedure. Nevertheless, the mode-coupling theory has been remarkably successful in applications to the KPZ equation as well as many other areas of the condensed matter theory, such as structural glass transitions [19], critical dynamics of magnets [20], binary mixtures, and others [21]. In all those fields, it has been found that the mode-coupling theory is capable of describing experiments in a quantitative manner. In Ref. [22], the mode-coupling equations for the KPZ equation were solved numerically to obtain the entire scaling functions in $1+1$ dimensions, and striking agreement with that obtained by direct numerical simulations was found [23]. Motivated by these facts, Frey *et al.* gave a systematic analysis of the mode-coupling approach using the field theoretical formulation of Langevin dynamics [24]. Doherty *et al.* [18] have shown that the mode-coupling equations become exact in the large N limit of a generalized N-component KPZ equation, which allows, in principle, a systematic approach to the theory beyond mode-coupling. In fact, there have been many analytical and numerical works involving the mode-coupling approach to the KPZ equation: Bouchaud and Cates [25] gave an approximate analytical solution by assuming simple exponential relaxation for each mode; Doherty *et al.* [18] used, instead, an ansatz based on the form of the scaling functions in $d=1$; Tu [26] solved numerically the mode-coupling equations by direct integration to determine the upper critical dimension of the KPZ; Moore *et al.* [27] obtained an explicit solution of the mode-coupling equations for $d>d_c=4$ with $z=2$. In their recent works, Colaiori and Moore [28,29] studied the mode-coupling approximation for the KPZ equation in the strong-coupling regime. They determined the upper critical dimension, dynamic exponents, and scaling functions by constructing an ansatz consistent with the asymptotic forms of the correlation and response functions [28], and derived, by using a saddle point analysis of the mode-coupling equations, exact results for the correlation function in the long-time limit—a limit that is hard to study using simulations [29]. So far, all the analytical works in the mode-coupling theory have been started by making an ansatz on the form of the scaling functions.

In the present work, we apply a self-consistent mode-coupling approach to the nonlocal KPZ equation [14] to investigate its dynamic scaling in the strong-coupling phase. In our discussion, the scaling function assumptions proposed in Ref. [28] are used. The corresponding values of the dynamic exponent that depend on nonlocal parameter ρ are calculated numerically for the substrate dimension $d=1$, $d=2$, and $d=3$, respectively.

The nonlocal KPZ equation proposed by Mukherji and Bhattacharjee [14] is

$$\frac{\partial h(\mathbf{x},t)}{\partial t}=\nu\nabla^2 h(\mathbf{x},t)+\eta(\mathbf{x},t)+\frac{1}{2}\int d\mathbf{x}'\,\vartheta(\mathbf{x}')\nabla h(\mathbf{x}+\mathbf{x}',t)\cdot\nabla h(\mathbf{x}-\mathbf{x}',t), \tag{3}$$

where the kernel function $\vartheta(\mathbf{x})$ has a short-range (SR) part $\sim\lambda_0\delta(\mathbf{x})$ and a long-range (LR) part $\sim\lambda_\rho x^{\rho-d}$. It was indicated, by simple scaling analysis, that both λ_ρ and λ_0 are relevant for $d<2$ at the Gaussian fixed point and the critical dimensions are given by $d_c=2+2\rho$ (for $\rho>0$) and $d_c=2$ (for $\rho<0$) for any nonzero λ_ρ. When $\rho>0$, the local KPZ theory is "unstable" under renormalization and a non-KPZ behavior is expected. For $2<d<2+2\rho$, only λ_ρ is relevant at the Gaussian fixed point [14,15]. The λ_ρ vs λ_0 phase diagram for the nonlocal KPZ equation is shown clearly in Ref. [14]. In the SR limit where the SR part of Eq. (3) dominates the LR part ($\lambda_0\neq 0$ and $|\lambda_\rho/\lambda_0|\ll 1$), Eq. (3) can smoothly go over to the local KPZ equation (1) in the case of $\lambda_\rho=0$. Our discussion in this paper will focus on the LR limit where the LR part of the nonlinear term in Eq. (3) dominates the SR part ($\lambda\rho\neq 0$ and $|\lambda_0/\lambda_\rho|\ll 1$), namely, only consider the LR axial fixed points [14]. It should be a noticeable question whether the following results do address the generalization of the single scaling regime of the one-dimensional KPZ equation, or there is a roughening transition, and the listed values are supposed to be those for the true strong-coupling rough phase beyond a nonequilibrium roughening transition. Janssen *et al.* [11] have argued that the mode-coupling approximation generically introduces spatially long-range correlations and therefore only addresses the former.

In the Fourier space, Eq. (3) becomes

$$h(\mathbf{k},\omega)=G_0(\mathbf{k},\omega)\,\eta(\mathbf{k},\omega)-\frac{1}{2}G_0(\mathbf{k},\omega)\int\frac{d\Omega}{(2\pi)}\frac{d^dq}{(2\pi)^d}\vartheta(|\mathbf{k}-2\mathbf{q}|)\mathbf{q}\cdot(\mathbf{k}-\mathbf{q})h(\mathbf{q},\Omega)h(\mathbf{k}-\mathbf{q},\omega-\Omega), \tag{4}$$

with $G_0(\mathbf{k},\omega)=1/(\nu k^2-i\omega)$ representing the bare response function. The correlation $C(\mathbf{k},\omega)$ and response functions $G(\mathbf{k},\omega)$ are defined by

$$C(\mathbf{k},\omega)=\langle h(\mathbf{k},\omega)h^*(\mathbf{k},\omega)\rangle, \tag{5}$$

and

$$G(\mathbf{k},\omega)=\delta^{-d}(\mathbf{k}+\mathbf{k}')\delta^{-1}(\omega+\omega')\left\langle \frac{\partial h(\mathbf{k},\omega)}{\partial \eta(\mathbf{k}',\omega')}\right\rangle, \tag{6}$$

where $\langle * \rangle$ denotes an average over noise $\eta(\mathbf{k},\omega)$. In the spirit of mode-coupling approximation [18,22,24], we derive the following self-consistent coupled equations for $G(\mathbf{k},\omega)$ and $C(\mathbf{k},\omega)$:

$$G^{-1}(\mathbf{k},\omega)=G_0^{-1}(\mathbf{k},\omega)+\int \frac{d\Omega}{(2\pi)}\frac{d^dq}{(2\pi)^d}\vartheta^2(|\mathbf{k}-2\mathbf{q}|)[\mathbf{q}\cdot(\mathbf{k}-\mathbf{q})][\mathbf{q}\cdot\mathbf{k}]G(\mathbf{k}-\mathbf{q},\omega-\Omega)C(\mathbf{q},\Omega), \tag{7}$$

$$C(\mathbf{k},\omega)=C_0(\mathbf{k},\omega)+\frac{1}{2}|G(\mathbf{k},\omega)|^2\int \frac{d\Omega}{(2\pi)}\frac{d^dq}{(2\pi)^d}\vartheta^2(|\mathbf{k}-2\mathbf{q}|)[\mathbf{q}\cdot(\mathbf{k}-\mathbf{q})]^2C(\mathbf{k}-\mathbf{q},\omega-\Omega)C(\mathbf{q},\Omega), \tag{8}$$

where $C_0(\mathbf{k},\omega)=2D|G(\mathbf{k},\omega)|^2$ is the bare correlation function. In the SR limit, $\vartheta(k)=\lambda_0$, Eqs. (7) and (8) can be reduced to the mode-coupling equations for the original KPZ equation [18,22–29]. In the strong-coupling limit, we look for the scaling solutions

$$G(\mathbf{k},\omega)=k^{-z}g(\omega/k^z),\quad C(\mathbf{k},\omega)=k^{-\Delta}n(\omega/k^z), \tag{9}$$

where $g(x)$ is a complex function and $n(x)$ is a real function. By substituting these scaling forms into Eqs. (7) and (8) and keeping only the leading terms in the limit $\omega,k\to 0$ while keeping ω/k^z finite, we find that the exponents Δ and z have to satisfy the following relation:

$$\Delta+z=4+d-2\rho. \tag{10}$$

As a result of the Galilean invariance of Eq. (3), there is the scaling exponent relation for χ and z [14,15],

$$\chi+z=2-\rho. \tag{11}$$

Therefore, from Eqs. (10) and (11), we obtain

$$\Delta=2\chi+d+z, \tag{12}$$

which, in fact, imply that the correlation function for the nonlocal KPZ equation has also the standard dynamic scaling form. In the LR limit, the scaling functions $g(x)$ and $n(x)$ satisfy the following equations:

$$g(x)^{-1}=-ix+I_1(x), \tag{13}$$

$$n(x)=|g(x)|^2I_2(x), \tag{14}$$

where $x=\omega/k^z$ and $I_1(x)$ and $I_2(x)$ are given by

$$I_1(x)=P\int_0^\pi d\theta\sin^{d-2}\theta\int_0^\infty dq\,q^{2z-3+2\rho}(\cos^2\theta-q\cos\theta)r_\rho^{-2\rho}r^{-z}\int dy\,g\left(\frac{x-q^zy}{r^z}\right)n(y), \tag{15}$$

$$I_2(x)=\frac{P}{2}\int_0^\pi d\theta\sin^{d-2}\theta\int_0^\infty dq\,q^{2z-3+2\rho}(\cos\theta-q)^2r_\rho^{-2\rho}r^{-(2x+d+z)}\int dy\,n\left(\frac{x-q^zy}{r^z}\right)n(y), \tag{16}$$

with $P=\lambda_\rho^2S_{d-1}/(2\pi)^{d+1}$, S_d is the surface area of the d-dimensional unit sphere, $r^2=q^2-2q\cos\theta+1$, and $r_\rho^2=4q^2-4q\cos\theta+1$. The dynamic exponent $z=z(d,\rho)$ can be obtained by requiring consistency of Eqs. (13) and (14) on matching both sides at an arbitrarily chosen value of x. In doing this, it is usual to make assumptions about the form of the scaling functions $g(x)$ and $n(x)$. Because of the nonlocality of Eqs. (13) and (14), the matching condition depends on the forms of the functions $g(x)$ and $n(x)$ for all x, so the assumptions need to be reliable for all x [28]. Before Colaiori and Moore's work [28], some assumptions had been proposed [18,25]. However, they all do not have the right large x asymptotic form [26]. In the case of $x\to\infty$, the integrals $I_1(x)$ and $I_2(x)$ are controlled by regions where $q\sim x^{1/z}$, so by simple power counting, we can obtain the following large x asymptotic behaviors for $g(x)$ and $n(x)$:

$$n(x)\sim x^{-1-\beta/z},\quad g_R(x)=\mathrm{Re}[g(x)]\sim x^{-1-2/z},\quad g_I(x)=\mathrm{Im}[g(x)]\sim x^{-1}, \tag{17}$$

with $\beta=4+d-2z-2\rho$.

It is convenient to discuss in Fourier space, in which Eqs. (13) and (14) can be written as [28]

$$\frac{\hat{g}_R}{|g|^2}(p)=\hat{I}_1(p), \tag{18}$$

$$\frac{\hat{n}_R}{|g|^2}(p)=\hat{I}_2(p), \tag{19}$$

where $\hat{I}_1(p)$ is the Fourier transform of the real part of $I_1(x)$ and $\hat{I}_2(p)$ is the Fourier transform of $I_2(x)$. They are expressed by

$$\hat{I}_1(p)=2\pi P\int_0^\pi d\theta\sin^{d-2}\theta\int_0^\infty dq\,q^{2z-3+2\rho}(\cos^2\theta-q\cos\theta)r_\rho^{-2\rho}\hat{g}_R(pr^z)\hat{n}(pq^z), \tag{20}$$

$$\hat{I}_2(p)=\pi P\int_0^{\pi} d\theta \sin^{d-2}\theta \int_0^{\infty} dq q^{2z-3+2\rho}(\cos\theta - q)^2 r_\rho^{-2\rho} r^{-(2x+d)}\hat{n}(pr^z)\hat{n}(pq^z). \tag{21}$$

Colaiori and Moore [28] proposed the scaling function assumptions for $\hat{n}$ and $\hat{g}$,

$$\hat{g}(p)=2C\theta(p)\exp(-|Dp|^{2/z}), \tag{22}$$

$$\hat{n}(p)=A\exp(-|Bp|^{\beta/z}), \tag{23}$$

with $D=1$ and $C=1/2$, A and B are parameters depending on d, z, and λ_ρ. It can be found that in x space, for large x, these assumptions give [28]

$$g(x)\simeq \text{const}\times x^{-1-2/z}+ix^{-1}, \tag{24}$$

$$n(x)\simeq \text{const}\times x^{-1-\beta/z}. \tag{25}$$

So the assumptions in Eqs. (22) and (23) have the right asymptotic behaviors that are consistent with Eq. (17). To match the large x behaviors of Eqs. (18) and (19), we can equivalently match them in the limit $p\rightarrow 0$, which actually means matching the most divergent terms on both sides of these two equations. In the large x limit, $|g(x)|^{-2}\simeq x^2$, so in the small p limit the left-hand sides in both Eqs. (18) and (19) are dominated by the terms $d^2\hat{n}/dp^2$ and $d^2\hat{g}_R/dp^2$ [28]. Accordingly, we have

$$(2-z)/z^2=\lim_{p\rightarrow 0}|p|^{2-2/z}\hat{I}_1(p), \tag{26}$$

$$AB^{\beta/z}\beta(\beta-z)/z^2=\lim_{p\rightarrow 0}|p|^{2-\beta/z}\hat{I}_2(p). \tag{27}$$

By performing the integrals and taking the limit $p\rightarrow 0$ in Eqs. (26) and (27), we can obtain the following coupled equations:

$$\frac{PAS'_d}{B}=\frac{d(2-z)}{\pi z^2}\,\frac{2^{2\rho}}{BI(B,z,\rho)}, \tag{28}$$

$$\frac{PAS'_d}{B^2}=\frac{\beta(\beta-z)}{\pi z^2}\,\frac{2^{(2z-\beta)/\beta+2\rho}}{\Gamma\left(\dfrac{2z-\beta}{\beta}\right)/\beta}, \tag{29}$$

where $S'_d=\int_0^{\pi}d\theta\sin^{d-2}\theta$, $\Gamma(u)$ is the Euler's gamma function, and $I(B,z,\rho)=\int_0^{\infty}ds(1-\rho-2s^2)s^{2z-3}\exp(-B^{\beta/z}s^{\beta}-s^2)$. In the simplest scenario, the parameter B can be taken to be $[2(2-z)]^{-1}$ (see Ref. [28] for details). So the values of the dynamic exponent z depending on the dimension d and nonlocal parameter ρ can be calculated by solving numerically Eqs. (28) and (29). We have calculated the values of z as a function of the parameter ρ in the case of dimension $d=1$, 2, and 3. The calculated results are shown in Fig. 1. For the sake of comparison, the results obtained by the DRG

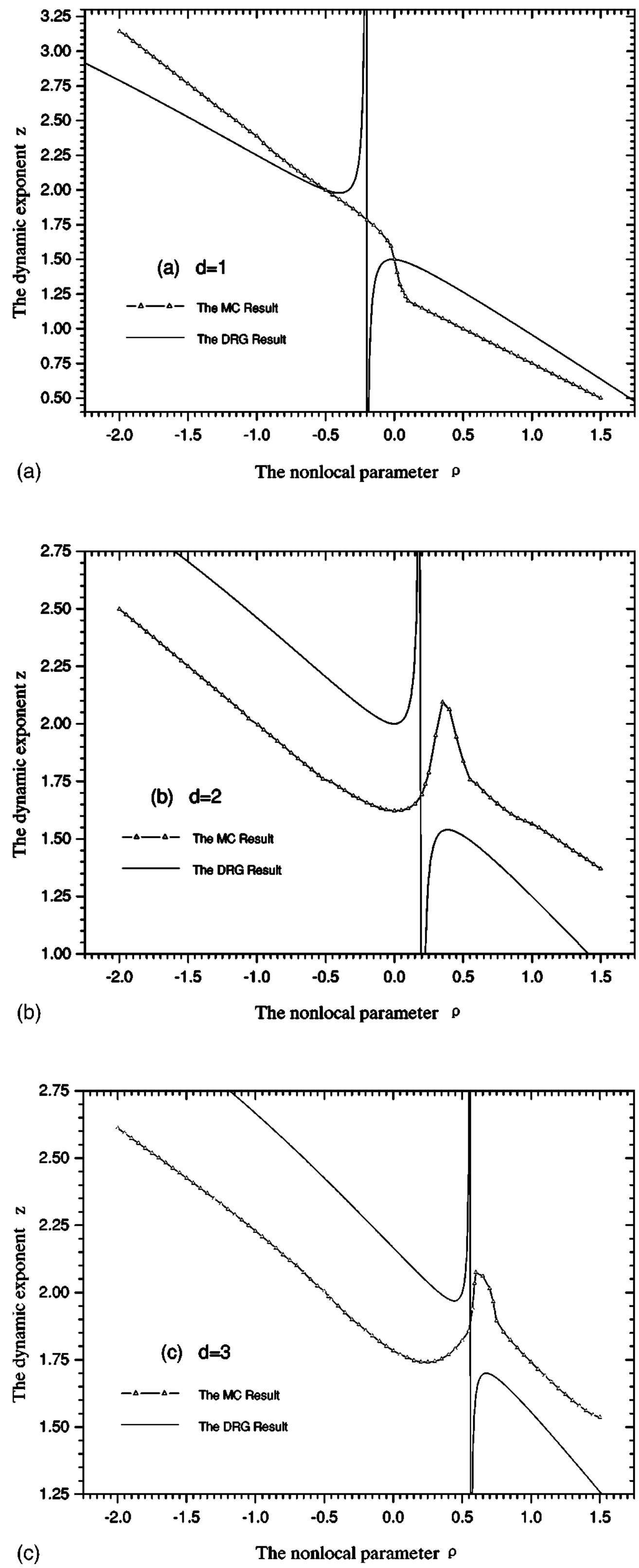

FIG. 1. The values of the dynamic exponent z as the function of nonlocal parameter ρ. (a), (b), and (c) are for $d=1$, 2, and 3, respectively, in which the solid lines express the DRG results and the $-\Delta-\Delta-$ lines express the mode-coupling (MC) results.

method in Ref. [14] are also shown in Fig. 1, where the dynamic exponent z is expressed by

$$z=2+\frac{(d-2-2\rho)(d-2-3\rho)}{d(2^{-\rho}+3)-6-9\rho}. \tag{30}$$

It can be seen that in the situation of $d=1$, although both the DRG theory and the mode-coupling approximation give the exact value $z=3/2$ for $\rho=0$, there exists a significant difference between the two kinds of approaches. In the DGR calculation, there is a divergence around $\rho=-0.4$, while in the mode-coupling approach, there seems to be no divergence but only an inflexion around $\rho=0$. In the case of $d=2$ and 3, the mode-coupling approximation in the present work gives the values $z=1.62$ $(d=2)$ and $z=1.78$ $(d=3)$ for $\rho=0$, which are identical to the results of Colaiori and Moore [28] as we expected. In addition, the result in $d=2$ is in good agreement with the values obtained from numerical simulation and with the direct numerical solution of the mode-coupling equation [26]. Thus we can believe that the scaling function assumptions made by Colaiori and Moore [28] are reasonable and the mode-coupling approximation employed here should give satisfactory results in the strong-coupling regime. In our mode-coupling approximation, the anomalous values of z occur at $\rho\sim0.3$ for $d=2$ and $\rho\sim0.55$ for $d=3$. One should see the analog of the lower critical dimension in these plots, namely, the critical $\rho_c=-1+d/2$; although the singularities occur at a larger value of ρ, the ρ_c seems to roughly correspond to the minima for the exponent z. It is not clear that there should not be a real divergence at some value of ρ, corresponding to the true divergence of the fixed point in the standard KPZ problem in two dimensions. Equation (30) addresses, on either side of the formal divergence, respectively (a) the single scaling regime below the critical dimension of the roughening transition (stable fixed point) and (b) the critical values at the transition (unstable fixed point). The divergent values of z in the DRG calculation are thought to be an artifact of the one-loop approximation [14]. The behavior of z as a function of ρ is poorly understood, and it is not clear at all which part of the plotted curves should be believed or considered artifacts of the mode-coupling approximation. As a matter of fact, it is actually not obvious at all which scaling behavior is addressed by the mode-coupling approximation. As mentioned above, since *a priori* no information is available about the size of the missing contributions, the mode-coupling theory constitutes an uncontrolled approximation which, we think, actually prevents us from giving more definite discussions on our results. Thus, further theoretical analyses of the mode-coupling approximation itself and the scaling function assumptions of Colaiori and Moore [28] will be very helpful and valuable.

In summary, we have applied a self-consistent mode-coupling approach to the nonlocal KPZ equation to study its scaling behavior in the strong-coupling regime. The values of the dynamic exponent z, as a function of the nonlocal parameter ρ, are calculated numerically in the substrate dimension $d=1$, 2, and 3. In our calculation, the scaling function assumptions of Colaiori and Moore [28], which have the right asymptotic behaviors, are used. Our derivation shows that the correlation and response function for the nonlocal KPZ equation also have the standard scaling forms except for different exponents.

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